Finally, in the solution (20) the physical parameters n and s must satisfy the equalities

$$\frac{n+1}{s}=\frac{m}{m-2},\ \alpha=0,\quad \frac{n+1}{s}=\frac{m}{m+2},\ \alpha=2.$$

NOTATION

T, temperature; r, coordinate; t, time; a, dimensional parameter; s, n, dimensionless parameters.

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EFFECT OF PERIODIC SYSTEM OF NARROW INCLUSIONS ON A PLANE STEADY TEMPERATURE FIELD

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Finding the complex potential of a plane temperature field perturbed by a periodic system of narrow inclusions reduces to solving a singular integrodifferential equation. The effect of cracks on an arbitrary periodic temperature field is considered.

1. Let a plane periodic (period $2a$) steady temperature field determined by the harmonic function $T_0(x, y) = \operatorname{Re} F(t)$ be perturbed by a $2a$-periodic system of narrow macroinclusions of a different material or cracks. For approximate formulation of the problem and its effective solution, we take the narrow inclusions as lines in the complex z plane. To be specific, we assume that the thermal conductivity of the inclusions k_0 is considerably less than that of the main medium (the body) k, i.e., $k_0 \ll k$.

Isolating in the z plane a band of width $2a$ $(-a \leqslant x \leqslant a)$, we denote the narrow inclusions present in the band, taken in any order, by Γ_n, n = 1, 2, ..., N. We denote the set of all the lines Γ_n by Γ, i.e., $\Gamma = \Gamma_1 + \ldots + \Gamma_N$.

The problem is to find the complex potential of the periodic temperature field perturbed by the inclusions, $W(z) = T + i\psi$; T is the temperature and ψ the current function.

We write W(z) as the sum of the potential of the temperature field of the homogeneous medium (without inclusions) F(z) and integrals of Cauchy type taken along the line Γ and all of the congruent lines, i.e., we write

$$W(z)=F(z)+\Phi(z),\ \Phi(z)=\frac{1}{4ai}\int\limits_{\Gamma}\omega(t)\operatorname{ctg}\frac{\pi(t-z)}{2a}dt. \qquad (1.1)$$

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The density $\omega(t)$ is determined using the boundary condition [1]

$$\delta \frac{\partial \psi}{\partial s} = T^- - T^+, \quad \delta(s) = \frac{h(s)}{\varepsilon}, \quad \varepsilon = \frac{k_0}{2kh_0}. \tag{1.2}$$

Here T^+ and T^- are the values of the temperature at the left-hand and right-hand edges of the inclusion; $2h_0h(s)$ is the width of the inclusion at cross section s; and h_0 = const.

Substituting Eq. (1.1) into Eq. (1.2) and using the Sokhotskii formula [2], we obtain a singular integrodifferential equation for

$$\omega(t) = T^+ - T^-, \tag{1.3}$$

in the form

$$|t'(s)| \frac{\omega(t)}{\delta(s)} = \mathrm{Re}\left[t'(s) \left(iF'(t) + \frac{1}{4a} \int_\Gamma \omega'(\tau) \operatorname{ctg} \frac{\pi(\tau - t)}{2a} dt \right) \right]. \tag{1.4}$$

Here s is an arbitrary monotonically increasing parameter such that, when s varies over the interval $[s^-, s^+]$, the point t(s) passes over the whole of the line Γ.

We assume that the limiting values of the temperature T^+ and T^- at each end of the lines Γ_n are equal and therefore it follows from Eq. (1.3) that the unknown function $\omega(t)$ must vanish at the ends of Γ:

$$\omega(t_n^-) = \omega(t_n^+) = 0. \tag{1.5}$$

Here t_n^- and t_n^+ denote the left-hand and right-hand ends of the line Γ_n, respectively.

The boundary condition (1.5) should be used in solving Eq. (1.4).

2. Direct solution of Eq. (1.4) may be achieved by various methods, depending on the form and properties of the inclusions. For example, if the line Γ consists of a set of N segments parallel to the same straight line, Eq. (1.4) may be transformed to a system of Fredholm equations of the second kind.

For brevity of exposition, we illustrate this for the case of isolated inclusions (N = 1). For N > 1, the discussion is similar.

Suppose that in the plane $z = x + iy$ $(i = \sqrt{-1})$ considered above there is a $2a$-periodic system of isolated (N = 1) inclusions (segments) lying along a straight line (the real axis ox); the inclusions are all of length $2b < 2a$. The equations of these segments may be written in the following form: $t = x + 2an, -b \leqslant x \leqslant b, b < a, y = 0$; n is an integer. The line Γ may be taken, for example, as the segment $-b \leqslant x \leqslant b$.

Introducing in Eq. (1.3) the expression $\gamma x_1 = \tan(\pi x/2a)$, where $\gamma = \tan(\pi b/2a)$, we obtain an equation of Prandtl type [3]:

$$\frac{\omega(x_1)}{\delta_1(x_1)} = \frac{1 + \gamma^2 x_1^2}{4a\gamma} \int_{-1}^{1} \frac{\omega'(\sigma)\, d\sigma}{\sigma - x_1} + f(x_1),$$

$$\delta_1(x_1) = \delta(x), \quad f(x_1) = \mathrm{Re}\,[iF'(x)] \tag{2.1}$$

Assuming that the function $(1 - x_1^2)^{1/2}/h_1(x_1)$, where $h_1(x_1) = h(x)$, has a first-order derivative continuous on the segment [−1, 1], and using the method of [3], Eq. (2.1) gives a regular equation for the unknown function $\omega(x_1)$:

$$\omega(x_1) = C_1 \sin\theta(x_1) + C_2 \cos\theta(x_1) - \frac{2\varepsilon}{\pi} \int_{-1}^{1} \omega(\sigma) K(\sigma, x_1)\, d\sigma + g(x_1), \tag{2.2}$$

$$K(\sigma, x_1) = \frac{2a\gamma}{\pi} \int_0^{x_1} \frac{\cos[\theta(\xi) - \theta(x_1)]}{\sqrt{1 - \xi^2}} \cdot \frac{\rho(\xi) - \rho(\sigma)}{\xi - \sigma}\, d\xi, \tag{2.3}$$

$$\rho(x_1) = \frac{\sqrt{1 - x_1^2}}{(1 + \gamma^2 x_1^2) h_1(x_1)}; \quad \theta(x_1) = 2\varepsilon \int_0^{x_1} \frac{d\sigma}{(1 + \gamma^2\sigma^2) h_1(\sigma)}, \tag{2.4}$$

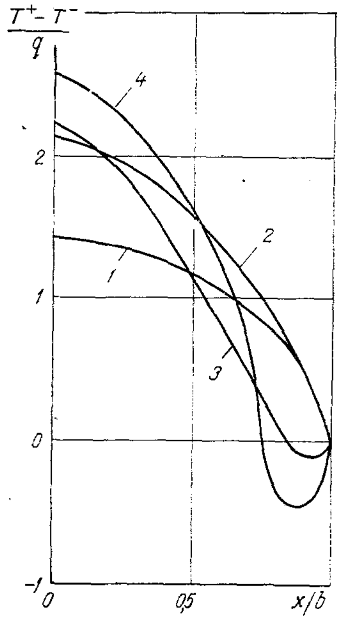

Fig. 1. Variation in dimensionless temperature drop along cracks for various values of b/a.

$$g(x_1) = \frac{2}{\pi}\int_0^{x_1} \frac{\cos[\theta(\xi)-\theta(x_1)]}{\sqrt{1-\xi^2}}\, d\xi \; \frac{2a\gamma}{\pi}\int_{-1}^{1} \frac{\sqrt{1-\sigma^2}\, f(\sigma)\, d\sigma}{(1+\gamma^2\sigma^2)(\sigma-\xi)} - \frac{4a\gamma}{\pi}\int_0^{x_1} \frac{f(\sigma)}{1+\gamma^2\sigma^2}\sin[\theta(\sigma)-\theta(x_1)]\, d\sigma. \tag{2.5}$$

The arbitrary constants C_1 and C_2 in Eq. (2.2) are determined from the boundary conditions $\omega(\pm 1) = 0$.

In each particular case, Eq. (2.2) is solved by any of the known methods, for example, by the method of successive approximations.

If the inclusions are nonconducting ($k_0 = 0$) or are in the form of very elongated ovals, i.e., $h_0 < 1$, and

$$h(x) = [1-\gamma^{-2}\,\mathrm{tg}^2(\pi x/2a)]^{1/2}\cos^2(\pi x/2a), \tag{2.6}$$

then $\varepsilon = 0$ or, correspondingly, the kernel $K(\sigma, x_1) \equiv 0$. Hence in these two cases the integral term in Eq. (2.2) vanishes and $\omega(x_1)$ is expressed, finally, in quadratures, using the relation

$$\omega(x_1) = C_1\sin\theta(x_1) + C_2\cos\theta(x_1) + g(x_1). \tag{2.7}$$

Determining $\omega(x_1)$ in accordance with Eq. (2.2), and then passing to the variable x and using Eq. (1.1), we find $\Phi(z)$.

Remark. In the case of a system of N inclusions parallel to the real axis ox in the band $|x| \leqslant a$, a discussion analogous to that above gives a system of N regular equations equivalent to Eq. (2.1).

3. As an illustration, consider the effect of a $2a$-periodic system of cracks ($k_0 = 0$, $\varepsilon = 0$) lying along a straight line, the real axis x ($t = x + 2an$, $|x| \leqslant b < a$; n is an integer), on a temperature field given by the complex potential

$$F(z) = q\exp(-i\alpha)\sin(\pi z/a); \quad q,\ \alpha - \mathrm{const}. \tag{3.1}$$

Substituting Eq. (3.1) into Eqs. (2.2)-(2.7) gives

$$\omega(x) = \frac{2q\sin\alpha}{\sqrt{1+\gamma^2}}\left[\left(1-\frac{1}{\sqrt{1+\gamma^2}}\right)\ln\frac{\sqrt{1+\gamma^2}-\gamma\sqrt{1-x_1^2}}{\sqrt{1+\gamma^2}+\gamma\sqrt{1-x_1^2}} + \frac{2\gamma\sqrt{1-x_1^2}}{1+\gamma^2x_1^2}\right], \quad x_1 = \frac{1}{\gamma}\,\mathrm{tg}\frac{\pi x}{2a}, \quad \gamma = \mathrm{tg}\frac{\pi b}{2a}. \tag{3.2}$$

For the function $\Phi(z)$, Eqs. (1.1) and (3.2) give

$$\Phi(z)=\frac{i2q\sin\alpha}{\sqrt{1+\gamma^2}}\left[\left(\frac{1}{\sqrt{1+\gamma^2}}-1\right)\left(\operatorname{arctg}\gamma z-\operatorname{arctg}\frac{\gamma\sqrt{z_1^2-1}}{\sqrt{1+\gamma^2}}\right)+\right.$$
$$\left.+\gamma\frac{\sqrt{1+\gamma^2}\,z_1-\sqrt{z_1^2-1}}{1+\gamma^2z_1^2}\right],\quad z_1=\frac{1}{\gamma}\operatorname{tg}\frac{\pi z}{2a}. \tag{3.3}$$

In Fig. 1, the variation in the temperature drop $T^+ - T^-$ (referred to q) along the cracks is shown for $\alpha = \pi/2$ and b/a = 0.25 (1), 0.5 (2), 0.75 (3), 0.9 (4) for $0 \leqslant x/b \leqslant 1$.

NOTATION

T^+, T^-, values of temperature T at the left-hand and right-hand edges of the inclusions; ψ, current function; W, complex potential of temperature field; k_0, thermal conductivity of inclusions; k, thermal conductivity of body; Γ_n, smooth line in complex z plane; Γ, piecewise-continuous line ($\Gamma = \Gamma_1 + \ldots + \Gamma_N$); $2h_0h(s)$, width of inclusion in section s (h_0 = const); 2b, length of inclusion; $2a$, period of complex potential W.

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SOLVING A SET OF DIFFERENTIAL EQUATIONS OF HEAT AND ELECTRICAL TRANSFER

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Methods are proposed in this article for solving the first boundary-value problem for a system of nonlinear differential equations for heat and electrical transfer in the general one-dimensional case.

1. It is known that the transfer of heat and charge in the media which possess thermoelectrical properties is governed by the equations of Maxwell, of heat conduction, and by the generalized Ohm's law. In the stationary case these equations can be written in the form [1, 2]

$$\operatorname{div}(\varkappa\nabla T)+\mathbf{JE}-\mathbf{J}\nabla(\alpha T)=0,$$
$$\mathbf{J}=\frac{1}{\rho}(\mathbf{E}-\alpha\nabla T),\ \operatorname{div}\mathbf{J}=0; \tag{1}$$

the solution of the equations under appropriate boundary conditions determines completely the fundamental characteristics of a thermoelement — the power generated W and the heat-flux density q:

$$W=-\int\mathbf{JE}\,dv,\ \mathbf{q}=-\varkappa\nabla T; \tag{2}$$

where T is temperature; E is the electric field intensity; $\alpha(T)$ is the coefficient of thermo-emf; $\varkappa(T)$ is the coefficient of thermal conductivity; and $\rho(T)$ is the coefficient of resistivity.

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